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en series converges rapidly, the sum may be found within any limits of precision. A few terms suffice to find the sum correctly within 7 or 8 decimals. The sum of the series without the squares would be exactly $\frac{1}{96}$.

GEOMETRY.

94. Proposed by EDMOND FISH, Hillsboro, Ill.

A tower $AB=a$, is surmounted by a flag pole $BC=b$. A point D is so taken in a line perpendicular to the foot of the tower that angle BDC is a maximum. Prove that AD is a mean proportional between AC and AB.

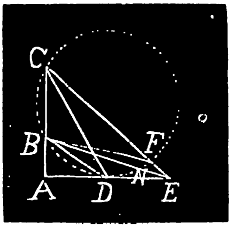
I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science, East Chester High School, Chester, Pa.; WALTER H. DRANE, A. M., Professor of Mathematics, Jefferson Military College, Washington, Miss.; C. A. JONES, Kosciusko, Miss.; E. R. GIBSON, Wayne, Neb.; and P. S. BERG, Larimore, N. D.

In order that $\angle BDC$ may be a maximum the line AD must be tangent to the circle through B, C at point D. If not let E be some other point in AD, and let EC cut the circle at F.

$\angle BFC > \angle BEC$. But $\angle BFC = \angle BDC$.

$\therefore \angle BDC > \angle BEC$. Q. E. D.

$\therefore AC : AD = AD : AB$.



II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Me.; W. L. HARVEY, Portland, Me., and the PROPOSER.

Describing a circle passing through B and C and tangent to AD, the angle BDC is measured by one-half the arc BC, whereas for any other point E the angle BEC is measured by one-half the difference of the arcs BC and FN, therefore by a smaller arc than BC. And since AD is tangent and ABC a secant, we have $AD^2 = AC \times AB$.

By the Calculus we may prove this fact thus: Denoting AD by x, we have

$$\tan BDC = \tan(CDA - BDA) = \frac{\tan CDA - \tan BDA}{1 + \tan CDA . \tan BDA}$$

$$= \frac{\dfrac{a+b}{x} - \dfrac{a}{x}}{1 + \dfrac{a(a+b)}{x^2}} = \frac{bx}{x^2 + a(a+b)}.$$

By differentiating, this expression is a maximum for $x^2 = a(a+b)$.

III. Solution by EDWARD R. ROBINS, Master of Mathematics, Lawrenceville School, Lawrenceville, N. J.; and COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Take A as origin, $AB=a$, $BC=b$, $AD=c$.

Equation of BD is $x/c + y/a = 1$.

Equation of CD is $x/c + y/(a+b) = 1$.

$$\therefore \measuredangle BCD = \tan^{-1}\frac{-bc}{c^2-a^2-ab},$$

which will be a maximum when $c^2-a^2-ab=0$, or $c^2=a(a+b)$. Q. E. D.

95. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

At each point of a parabola is described the rectangular hyperbola of a four-pointic contact; prove that the locus of the center of the hyperbola is an equal parabola.

Solution by the PROPOSER.

The curve having four-pointic contact with the parabola $y^2=4ax$......(1)

is $y^2-4ax-\lambda(yy'-2ax-2ax')^2=0$......(2),

or, $-4\lambda a^2x^2+4a\lambda y'xy+(1-\lambda y'^2)y^2-(4a+8a^2\lambda x')x+4a\lambda x'y'y-4a^2\lambda x'^2=0$..(3).

If this be an equilateral hyperbola,

$4\lambda a^2=1-\lambda y'^2$, or $\lambda=1\div(y'^2+4a^2)$......(4).

Substituting this in (2) and reducing,

$ax^2-y'xy-ay^2+(4a^2+y'^2+2ax')x-x'y'y+ax'^2=0$..... (5).

The center of this is given by

$x=-(y'^2+8a^2)/4a$....(6), $y=y'$....(7). (x', y') being on (1), $y'^2=4ax'$....(8).

Eliminating x', y' from (6), (7), and (8), we have the required locus,

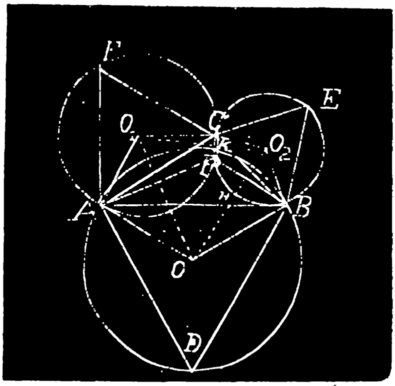
$y^2=-4a(x+2a)$......(9).

96. Proposed by W. F. BRADBURY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass.

Isosceles triangles are constructed externally on the three sides of a triangle as bases, with the angles at the bases each 30°. The triangle formed by joining the remote vertices (the 120° vertices) of these isosceles triangles is equilateral. [Geometric—not Trigonometric—solution.]

Solution by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.

The vertices O, O_1, O_2, of the isosceles triangles are the centers of equilateral triangles described on the sides of ABC. Circumferences passed about these triangles intersect in a point, P.



Let P be the intersection of the two circles, AFC and CEB. Join AP, BP, and CP. Since $APCF$ is inscribed, $\angle F + \angle APC=180°$. But $\angle F=60°$.

$\therefore \angle APC=120°$.

Similarly, $\angle CPB=120°$.

$\therefore \angle APB=120°$; and $\angle APB + \angle D=180°$.

$\therefore$ $APBD$ is inscribed, and P is in the circumference of ADB. Q. E. D.

Lines that join the centers of intersecting circles bisect the common